The M-algorithm on the Detection of CPM Schemes on the Minimum Euclidian Distance Upper Bound

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***Abstract* — The impact of the M-algorithm on continuous phase modulation (CPM) detection is analyzed when replacing the common Viterbi algorithm (VA) on the maximum likelihood sequence detection (MLSD) block. The algorithm is presented as a reduced complexity sequence detection algorithm in which not all transitions are propagated, being only retained a small subset of paths. A rule is found concerning the lower limit of the mandatory number of paths to retain, using simulations with additive white gaussian noise (AWGN) when detecting simple catastrophic schemes and two optimum gain schemes, in the sense they are two of the rare coincident with their respective minimum Euclidean upper bounds. The M-algorithm proves near optimum performance for very low ratios of number of traced states per total number of states.**

I. INTRODUCTION

Continuous phase modulation (CPM) signals have constant amplitude, so their insensitivity to non-linear amplitude amplification makes them useful in systems penalised by that problem. Phase continuity allows good spectral performance and implies a code gain due to the inherent memory effect. These properties have motivated the widespread use of GMSK (gaussian minimum shift keying). The implementation of others CPM schemes more spectrally and power efficient was restrained owing to excessive detection complexity.

The problem of CPM detection poses two main problems: how to obtain all the transitions metrics and how to search the most probable sequence of states. The optimum detector usually demands a very large bank of matched filters to obtain all phase transitions metrics and. Moreover, the number of phase states on the Markov chain that must be detected afterwards can also be very large, even without channel coding [1]. This paper addresses the later problem.

II. CPM FORMATTING AND PERFORMANCE

Every CPM signals can be expressed in the form:

$$s(t,\boldsymbol{\gamma}) = \sqrt{2E_s/T_s}\,\cos(\omega_c t + \varphi(t,\boldsymbol{\gamma}) + \varphi_0)\,. \qquad (1)$$

The carrier frequency is f_c, where $\omega_c = 2\pi f_c$, φ_0 is the arbitrary initial phase and E_s is the energy per symbol, related with the bit energy by $E_s = \log_2(M)\cdot E_b$. Channel symbols are $\gamma_i \in \{\pm1, \pm3, \cdots, \pm(M-1)\}$, forming the M-ary sequence $\boldsymbol{\gamma}$. Each symbol γ_i carries $\log_2(M)$ bits as a result of a natural mapping of the information bits stream $\boldsymbol{\alpha}$. The information carried by N_S channel symbols is keyed in signal's phase:

$$\varphi(t,\boldsymbol{\gamma}) = 2\pi\, h \sum_{i=0}^{N_s} \gamma_i\, q(t - iT_s)\,. \qquad (2)$$

A constant modulation index, $h=p/q$, is considered, where p and q are integers with no common factors. (h is rational in order to have a finite number of phase states.) Phase transition pulse shape, $q(t)$, affects phase transitions shape during L symbols. However, its effect remains until the end of the transmitted sequence. $q(t)$ is defined by the frequency pulse $g(t)$: $q(t) = \int_{-\infty}^{t} g(\tau)\,d\tau$. The normalisation $q(t) = \int_{0}^{\infty} g(\tau)\,d\tau = 1/2$ is applied so that the maximum phase transition during a symbol time, T_s, is $h\cdot(M-1)\cdot\pi$. Different frequency pulses define different CPM families. The most common are: LREC, LRC (L is the variable mentioned above) and GMSK [1,2]. LREC is defined by $g(t)=\text{rect}[t/(LT_s)]/2$, where $\text{rect}(t)=1$ for $-1/2<|t|<1/2$ and zero elsewhere. Schemes with 1REC pulses are also known as CPFSK (continuous phase frequency shift keying). A smother $g(t)$ can improve the spectral efficiency of schemes with LREC pulses, an example is the referred LRC which has a raised cosine pulse shaping.

In order to evaluate CPM power performance one uses the *minimum normalised squared euclidean distance* (MNSED) between two signals transporting sequences $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma}'$:

$$d^2_{\min}(\boldsymbol{\gamma},\boldsymbol{\gamma}') = 1/(2E_s) \min_{\gamma,\gamma':\gamma\neq\gamma'} \int_0^{\infty} \left[s(t,\boldsymbol{\gamma}) - s(t,\boldsymbol{\gamma}')\right]^2 dt\,. \quad (3)$$

Bit error rate (BER) is given by (e.g. [1])

$$P_b \approx C\cdot Q\left(\sqrt{d^2_{\min}\frac{E_b}{N_0}}\right) \approx Q\left(\sqrt{d^2_{\min}\frac{E_b}{N_0}}\right). \qquad (4)$$

C is a constant ≈ 1 for most schemes (and 2 for MSK). $Q(x)$ is the area under the unit variance gaussian distribution in $[x, \infty]$. Power efficiency comparisons can be made from (4) merely by $d^2_{\min}$ knowledge or converting it to a gain relative to MSK, being $G = 10\cdot\log_{10}(d^2_{\min}/2)$ [dB].

Bandwidth is usually given in terms of $B_\varepsilon T_b$ where B_ε is the bandwidth that enclosures ε% of all transmitted power. $T_b = T_s/\log(M)$ is the bit interval; bit rate is

R_b=1/T_b. For MSK $B_{99.0}T_b$=1.2. Spectrum efficiency is thereby ζ=1/($B_\varepsilon T_b$)=R_b/B_ε. By reducing h phase transitions get smother, tightening bandwidth, but that also forces MNSED to decrease due to the greater similitude among transitions during each T_s interval. A greater M enhances simultaneously spectrum and power behaviour at a cost of boost in complexity.

III. CONSTRAINED COMPLEXITY SEQUENCE DETECTION

The Markov property of CPM allows its description by a trellis with a certain number of states. To obtain all Ξ transition metrics the optimum CPM receiver requires 2Ξ matched filters (or equivalent correlators). The optimum receiver is depicted in Figure 1.

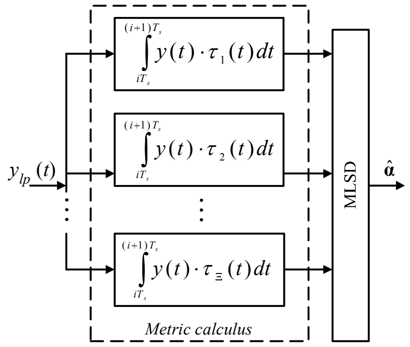


Figure 1: Optimum receiver with AWGN.

Metrics have to be calculated for all in phase and quadrature transitions, respectively $\tau_{I,b} \in \{\tau_{I,1}, \tau_{I,2}, \cdots, \tau_{I,\Xi}\}$ and $\tau_{Q,b} \in \{\tau_{Q,1}, \tau_{Q,2}, \cdots, \tau_{Q,\Xi}\}$. Considering AWGN, $n(t)$, with bilateral power spectral density N_0/2 W/Hz, after baseband conversion one gets the signal $y(t)=s(t,\gamma)+n(t)$. Metrics for b= 1, 2, ···, Ξ, are given by

$$\Lambda_i(b) = \int_{T_s} y_{I,i}(t)\tau_{I,b}(t)\,dt + \int_{T_s} y_{Q,i}(t)\tau_{Q,b}(t)\,dt = \Lambda_{I,i}(b)+\Lambda_{Q,i}(b). \quad (5)$$

In more detail, for the same b, the branch metrics are

$$\Lambda_{I,i}(b) = \int_{iT_s}^{(i+1)T_s} y(t)\cdot\cos[2\pi h\gamma_b q(t)]dt \quad (6a)$$

$$\Lambda_{Q,i}(b) = \int_{iT_s}^{(i+1)T_s} y(t)\cdot\sin[2\pi h\gamma_b q(t)]dt\ . \quad (6b)$$

Finally, having all the metrics, the problem is solved by a maximum likelihood sequence detector (MLSD). The detection complexity of CPM schemes is measured in terms of Ξ and the total number of states, being that number $S=q\cdot M^{L-1}$, for even p and $S=2q\cdot M^{L-1}$ for odd p. On full response systems (L=1), S corresponds to the number of physical phase states (Figures 2 and 3). The number of phase transitions is therefore $\Xi=S\cdot M$. For this reason the number of 2Ξ filters becomes intolerable for high M and/or weak h.

Each transition has the incremental metric

$$\Lambda_i(b) = \|y(t,\gamma_i)-\tau_b(t)\|^2 = \|y(t,\gamma_i)\|^2 + \|\tau_b(t)\|^2 - 2\langle y(t,\gamma_i),\tau_b(t)\rangle \quad (7)$$

As a result, MLSD must search for the sequence having maximum cumulative metric given by the inner product

$$\Lambda_i(b) = \langle y(t,\gamma_i),\tau_b(t)\rangle\ . \quad (8)$$

The MLSD algorithm must so search for the sequence having maximum metric and not the minimum. By defining the detection complexity as the number of paths being traced in the trellis, the complexity of the VA is equal to S, which is typically large.

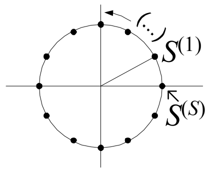


Figure 2: Physical phase states for full response CPM.

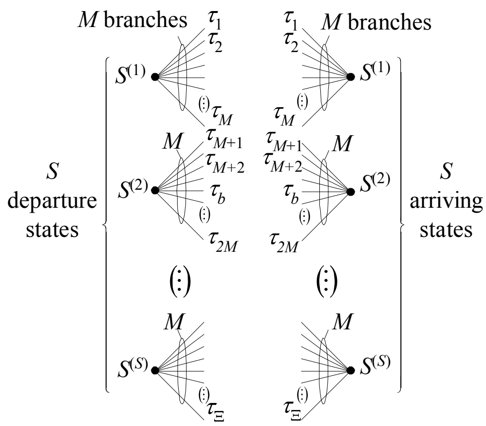


Figure 3: Trellis for M-ary CPM.

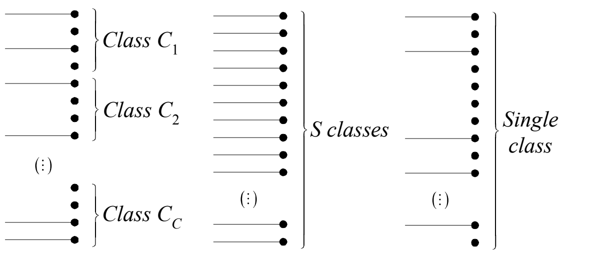
Having all the metrics the problem remains on search of the most probable sequence of phase transitions. For that purpose the Viterbi Algorithm (VA) is widely used. It performs maximum likelihood sequence detection (MLSD) but its complexity can limit its use. The problem of performing MLSD under given complexity constrain results in a family of optimal detectors is presented in [3]. Complexity constrained MLSD can be described by the *search algorithm*, which separates the S states into C classes. Hence, every class contains S/C states. Within each class some paths are discarded at each symbol decision. Only those paths closer to the received signal in the Euclidean distance sense are to be retained. B is the number of paths chosen to remain in competition inside each class. This algorithm and its variables are denoted by SA(B, C). One can recognize the VA as being the particular case SA(1, S) – Figure 4. One advantage of Viterbi detection over sequential detection is that the number of states is constant on each symbol time, against a variable number on sequential decoding [4]. The entire SA(B,C) family performs MLSD over the AWGN channel in the sense that increasing SNR conducts to arbitrary low error probability of choosing a wrong sequence, if no inter-symbol interference (ISI) exists [3].

Whenever one search is conducted inside partitions the algorithm is usually named reduced state sequence detection (RSSD). RSSD can be denoted by SA(1, *C*). At the beginning of each interval *MB* transitions emerge, but only $N_p=BC$ paths are stored as initial states of the next iteration and N_p is always $<S$. So, SA($B=N_p$, 1) conducts to the best performance since it is the least constrained situation. A lot of attention has been devoted recently to this sub-family of algorithms (e.g. [3, 4] and references therein).

The M-algorithm is a particular case of this RSSD family, corresponding to the case SA(*B*, 1), being M=*B* (notice the difference for *M*, the *M*-arity). This is the algorithm evaluated on this paper due to the great amount of simplicity it can bring to CPM detection and for belonging to the family of best performance (Figure 4).

SA(*B*,1) is the optimum algorithm: despite having the same performance of SA(*S*,1) (i.e. the VA), SA(*B*,1) minimizes the probability of a first error event since the best metric paths are always propagated and not one for each state as in the VA, possibly losing some paths with a larger metric. (In general the number of retained paths is not limited to *S*; but can be has large as *BC*.)

In the limit of *B*=1 one gets decision feedback (DF), i.e., SA(1, 1); only one path is traced for the sequence detection. It corresponds to the case of less computational weight but the probability of losing the correct path is the highest one. It can only be used during periods when the channel is known to have reduced noise on a strategy of a variable M-algorithm, being *M* controlled by noise power estimation.



a) SA (*B*=2,*C*) b) SA(*B*=1,*S*) / VA c) SA (*B*=5,*C*=1) / *M*-alg.

Figure 4: The M-algorithm as a sub-class of the SA(*B*,*C*).

IV. TEST SCHEMES

From [2,6,7] it is possible to present the power and spectrum characteristics of some schemes with *M*=2, 4 and 8 in Table 1 (MSK on the first line) and also for two optimum full response mono-*h* CPM schemes. Those schemes of *h*=0.45=9/20 are the best 4-ary and 8-ary CPFSK schemes in terms of power gains within the region of useful spectrum efficiencies holding an acceptable number of states (*S*=40).

Values for positions of Table 1 where "(a)" is found are not available in literature and can be found in Section V.

Table 1: Characteristics of 1REC CPM schemes

h	M	S	$B_{99.0}T_b$	d^2_{min}	G [dB]	Ξ	2Ξ
1/2	2	4	1.20	2.0	0	8	16
	4	4	1.30	2.0	0	16	32
	8	4	1.55	3.0	1.76	32	64
	16	4	(a)	(a)	(a)	64	128
9/20	4	40	1.18	3.60	2.56	160	320
	8	40	1.40	5.40	4.31	320	640

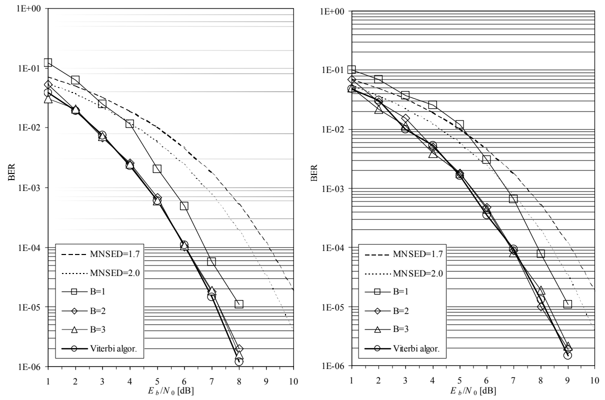
In order to research the behaviour of the receiver operating with the Walsh space projections as a function *M* we have used the *h*=1/2 full response *M*-ary schemes presented in Table 1, taking advantage of their very low number of states (*S*=4). Those simple schemes happen to be catastrophic, that is, their MNSED has a local mean for the used *h*=1/2, being the real d^2_{min} very distant from its upper bound [1,2]. This references also contain an evolution of MNSED obtained by simulation as a function of the modulation index *h*. Analysing the interval for $h\in[0.1\ \ 0.5]$ (which corresponds to the interesting schemes in terms of bandwidth - smother transitions) one finds *h*=0.33 and *h*=0.5 corresponding to catastrophic schemes for both for *M*=4 and for *M*=8 (as for *M*=2, i.e, MSK). Apart *h*=0.33, in that interval the curves of upper bound and simulated MNSED are coincident rising very fast until d^2_{min}=3.6 at *h*=0.45=9/20. Then MNSED drops to d^2_{min}=2 at *h*=1/2, as for MSK. For *M*=8 the study of the same authors shows rather irregular behaviour in terms of MNSED for $h\in[0.1\ ,\ 0.5]$. Four catastrophic schemes are found, being one at *h*=1/2, just as for *M*=2 and *M*=4. Moreover, only two different *h* conduct to optimal MNSED, i.e., MNSED coincident with its upper bound. The selected *h*=9/20 schemes are optimum in the sense that few schemes have a minimum Euclidean distance equal the respective upper bound.

V. RESULTS

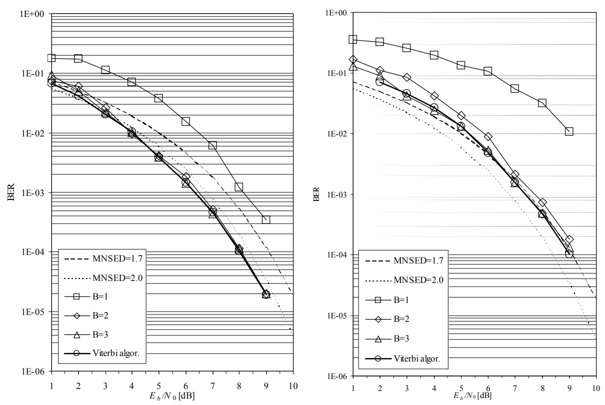
Performance is determined in terms of bit error rate (BER) for detection by means of Monte Carlo simulation under AWGN. (We use the letter *B* of the general SA(*B*,*C*) and not the one on the algorithms name.) For better power gain assessment all figures include the BER curve for ideal antipodal modulation ($d^2_{min}=2$) and the BER curve associated to $d^2_{min}=1.7$, proposed by [8] to describe real MSK power performance.

For the *h*=1/2 schemes (Figure 5) it is found a similar pattern for bit error rate performance as a function of the number of states to propagate on the trellis, *B*. An increase on the modulation *M*-arity, requires a rising on that path memory number. Is can be seen that for all tested schemes *B*=*M* assures near optimum performance. For *B*>*M* no significant gains are detected. Inversely, Further decreasing *B* implies an abrupt decay on performance, being this effect more important for high *M* schemes.

From Figure 6 one can conclude that the rule $B\geq M$, found out for the simple but catastrophic schemes, can be extrapolated to the interesting *h*=9/20 schemes. Both *h*=9/20 *M*=8 and *M*=4 schemes (having *S*=40) can be detected using *B*=5, keeping a BER performance as close as 0.2 dB from the optimum detection curve. In addition, the curves for the optimum receivers confirm the expected gains showed in Table 1 for these *h*=9/20 schemes: 2.6 dB for *M*=4 and 4.3 dB for *M*=8.

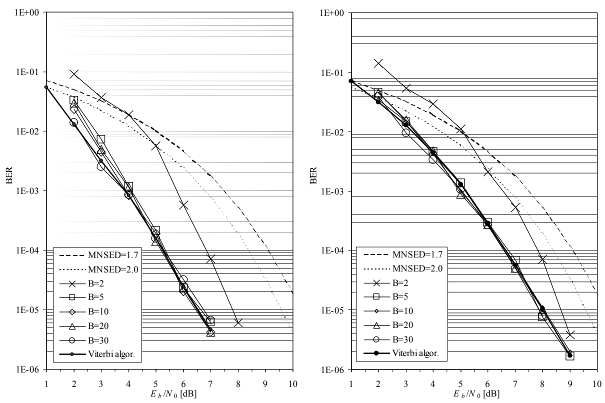


(a) *M*=16 (b) *M*=8



(c) *M*=4 (d) *M*=2 (MSK)

Figure 5: Detection of 1REC, *h*=1/2, for varying *B*, with AWGN.



(a) *M*=8 (b) *M*=4

Figure 6: Detection of 1REC, *h*=9/20, for varying *B*.

The results found in this paper are settled upon the theoretical study of Aulin [3] for general MLSD. The authors of [9] cite unpublished results by this author congruent with the ones encountered here.

VI. CONCLUSION

It has been found that high gain CPM schemes can be quasi-optimally detected when propagating on its trellis a number of paths as small as the modulation *M*-arity. Thus, the amount of complexity on the MLSD block for both selected optimum CPM schemes could be reduced by a factor of 10.

The results provided by this paper permitted to combine the M-algorithm with other complexity reduction techniques for CPM such as [10,11] in order to achieve a quasi-optimum very low complexity CPM receiver [12,13].

ACKNOWLEDGEMENT

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