Dual-Lattice-Aided MIMO Detection for Slow Fading Channels

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Abstract— **This paper presents a lattice detection strategy for spatial multiplexing (SM) which takes advantage of a pre-processing stage based on the geometric relations between the points in the primal lattice and the ones in the dual lattice. The first part of the paper clarifies this geometric relationship that will be exploited later on in the design of a pre-processing stage for the proposed receiver. This pre-processing finds a set of successive minima in the dual lattice, and is only required at each channel update. The subsequent symbol detection algorithm exclusively involves a linear transformation (the pseudo-inverse) in order to generate a list of candidate solutions for the underlying closest vector problem (CVP). The receiver outperforms ordered successive interference cancellation and, in the low signal-to-noise ratio (SNR) regime, also outperforms lattice-reduction-aided receivers at the expense of a "true" sphere-decoder that runs only once per channel update, and not for each received vector.**

Keywords: *MIMO detection, spatial multiplexing, closest vector problem (CVP), dual lattice, hyperplanes.*

I. INTRODUCTION

The detection of a received vector in a multiple-input multiple-output (MIMO) system is equivalent to solving a closest vector problem (CVP) in a lattice. Geometric lattices, defined as discrete subgroups in $\mathbb{R}^n$, are simple to define mathematically, and have an apparent geometrical simplicity. However, lattices are closely related with many problems having NP-hard algorithmic complexity, one of them being the CVP [1]. The CVP consists of finding the lattice point which is the one at the shortest distance from a given off-lattice target point. The advent of MIMO triggered a series of re-discoveries and novel uses of ideas previously studied for abstract lattices in algorithmic number theory and others which were already known in single-input single-output (SISO) contexts: i) the Babai nearest plane algorithm [2] (pp. 40-44), known in MIMO as vertical Bell Labs layered space-time (VBLAST) detection or ordered successive interference cancelation (OSIC) [3]), ii) the Lenstra-Lenstra-Lovász (LLL) algorithm [4] used in lattice-reduction-aided (LRA) receivers [5], or iii) tree-exploration by means of sphere decoding (SD) are examples of this [1].

The simplest detection techniques for the CVP in MIMO are those based on zero-forcing (ZF) or the minimum mean square error (MMSE). These receivers suffer from noise enhancement, which has only been fully characterised very recently [6]. OSIC attains a better performance than any of the linear receivers and can be used in conjunction with either ZF or MMSE filters when detecting each layer. The traditional implementation of OSIC has a complexity $\mathcal{O}(n^4)$, where n is the dimension of the lattice (which in MIMO corresponds to the number of receive antennas), though there exist implementations that only require $\mathcal{O}(n^3)$ (see [7] (p.39) and references therein). The performance of SIC techniques only depends on the length of the Gram-Schmidt vectors associated with the basis of the lattice. Capitalising on that, the authors in [8] proposed an OSIC algorithm with $\mathcal{O}(n^3)$ complexity based on the geometric insights offered by the dual lattice. LRA receivers usually implement LLL reduction at the pre-processing stage. LLL, which attains full diversity [9], is known to have complexity $\mathcal{O}(n^4)$ for integer matrices [10], however, it has been proven in [5] that in real valued MIMO some channels may lead to an unbound number of operations (though the probability of that happening is close to zero). It is worth mentioning that the application of LLL to the dual lattice proved beneficial to LRA receivers [9]. Optimum detection performance can be achieved by sphere decoding (SD), at the expense of an exponential complexity [11], however, for the typical number of antennas involved in wireless MIMO, the complexity is affordable and in the proposed receiver SD is only needed once per channel update. This paper proposes a MIMO detector which makes use of a "true" sphere decoder, not to solve a CVP, but rather to capture all the vectors of the dual lattice that are inside a hypersphere centred at the origin of the lattice (hence the reference to a "true" SD process, because it applies SD in its simplest conception, as in [12]).

The elegant geometric relationship between the primal and the dual lattice is often overlooked in MIMO literature and, after introducing some definitions in Section I, a brief tutorial on that relationship is presented in Section II. With a geometric perspective of the CVP in mind, the proposed receiver is presented in Section III.

II. MIMO AND LATTICES

A. System Model for Spatial Multiplexing

In MIMO SM with N_T transmit antennas and N_R receive antennas (with $N_R \geq N_T$), the relationship between the transmitted vector $\mathbf{x}_c = [x_{c,1}, x_{c,2}, \ldots, x_{c,N_T}]^T \in \mathbb{C}^{N_T \times 1}$ and the received vector $\mathbf{y}_c = [y_{c,1}, y_{c,2}, \dots, y_{c,N_R}]^T \in \mathbb{C}^{N_R \times 1}$ is modelled in the baseband as

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{x}_c + \mathbf{n}_c , \qquad (1)$$

where $\mathbf{H}_c \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix with its entries h_{ij} representing the complex coefficient associated with the SISO link between he i^{th} receive antenna and the j^{th} transmit antenna, and, in the case of Rayleigh flat fading channel, h_{ij} are taken from a zero-mean circularly symmetric complex Gaussian distribution with unitary variance (i.e., variance 1/2 in both the real and imaginary components). Furthermore, the noise added to each entry of the received vector is modelled by the column vector $\mathbf{n}_c = [n_{c,1}, n_{c,2}, \dots, n_{c,N_R}]^T \in \mathbb{C}^{N_R \times 1}$ with independent circularly symmetric complex Gaussian random variables taken with zero mean and variance σ_n^2 (corresponding to a variance $\sigma_n^2 / 2$ in both real and imaginary components). The energy of the complex transmitted symbols is assumed to be $E\{x_{c,i}^2\} = 1$. It is not difficult to prove that by stacking the real and complex parts of the vectors (respectively denoted by $\Re$ and $\Im$), and by appropriate construction of a modified channel matrix, the problem can equivalently be described by means of real variables as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \Leftrightarrow$$

$$\Leftrightarrow \begin{bmatrix} \Re(\mathbf{y}_c) \\ \Im(\mathbf{y}_c) \end{bmatrix} = \begin{bmatrix} \Re(\mathbf{H}_c) & -\Im(\mathbf{H}_c) \\ \Im(\mathbf{H}_c) & \Re(\mathbf{H}_c) \end{bmatrix} \begin{bmatrix} \Re(\mathbf{x}_c) \\ \Im(\mathbf{x}_c) \end{bmatrix} + \begin{bmatrix} \Re(\mathbf{n}_c) \\ \Im(\mathbf{n}_c) \end{bmatrix} \qquad (2)$$

with all vectors now real (hence, the "*c*" subscripts in the variables are dropped from now on). The real symbols in each dimension are taken from an alphabet $\mathcal{A}$, with $\sqrt{M}$ symbols, in the case of *M*-QAM modulation in each antenna.

B. Lattices

A real lattice Λ can be mathematically defined in more than one way [13]; These geometric lattices, defined as discrete subgroups in $\mathbb{R}^n$, are most times defined by a generator matrix constructed from a set of vectors (i.e., a basis) which spans the lattice:

$$\Lambda = \left\{ \mathbf{y} \in \mathbb{R}^n : \mathbf{y} = \sum_{i=1}^{n} \mathbf{h}_i x_i = \mathbf{H} \cdot \mathbf{x} , \quad x_i \in \mathbb{Z} \right\} . \qquad (3)$$

As defined in (3), the lattice will result from integer combinations of the columns of the generator matrix $\mathbf{H}$. (Some authors prefer spanning the row space of the matrix.)

The *Gram matrix* of a real lattice is defined by $\mathbf{G} = \mathbf{H}^T \mathbf{H}$. By construction, the Gram Matrix contains all the possible inner products between all the generator vectors: $g_{ij} = \left\langle \mathbf{h}_i, \mathbf{h}_j \right\rangle$; in particular, the diagonal elements are the squared norms $\left\| \mathbf{h}_i \right\|^2$. This fact implies that $\mathbf{G}$ is symmetric and positive definite, and defines a positive definite quadratic form.

Any lattice may be defined by an infinite number of bases, but given a certain basis of a lattice, the *fundamental region* that is associated with that basis is defined as

$$\mathcal{R}(\mathbf{H}) = \left\{ \mathbf{H}\mathbf{x} : 0 < x_i < 1 \right\} . \qquad (4)$$

When $\mathbf{H}$ is square and non-singular, the lattice is full-rank, and the volume of the lattice (the volume of $\mathcal{R}$) is

$$\mathrm{vol}(\Lambda) = \left| \det(\mathbf{H}) \right| . \qquad (5)$$

However, for a rectangular $\mathbf{H}$, the following more general definition is required:

$$\mathrm{vol}(\Lambda) = \sqrt{\det(\mathbf{H}^T \mathbf{H})} = \sqrt{\det(\mathbf{G})} . \qquad (6)$$

The volume of the lattice is an invariant of the lattice, i.e., it is independent of the choice of basis.

III. The Geometry of the Dual Lattice

Every lattice (said to be the *primal* lattice) has a *dual lattice*, also known as the *polar lattice* or, more commonly, as the *reciprocal lattice*. Note that these names were already in use in the early 70's [14] (p.24). Since then, the name polar has fallen into disuse, though reciprocal can still be found in some literature. The dual lattice is traditionally defined for real lattices, though the definition has also been extended to complex lattices [8]. Given the intuitive geometrical interpretation that is possible in the real domain, the dual lattice is usually defined for real lattices as

$$\Lambda_D = \left\{ \mathbf{z} \in \mathbb{R} : \left\langle \mathbf{z}, \mathbf{x} \right\rangle \in \mathbb{Z} \quad , \forall \mathbf{x} \in \Lambda \right\} . \qquad (7)$$

The dual lattice can also be expressed in terms of the dual basis $\mathbf{H}^{(D)}$ as

$$\Lambda_D = \left\{ \mathbf{z} \in \mathbb{R} : \mathbf{z} = \underbrace{\left(\mathbf{H}^+ \right)^T}_{\mathbf{H}^{(D)}} \mathbf{x} , \quad \mathbf{x} \in \mathbb{Z}^n \right\} . \qquad (8)$$

where $\mathbf{H}^+$ is the Moore-Penrose pseudo-inverse

$$\mathbf{H}^+ = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H . \qquad (9)$$

Hence,

$$\mathbf{H}^{(D)} = \mathbf{H}^T \left(\mathbf{H}^T \mathbf{H} \right)^{-1} . \qquad (10)$$

In fact, for $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{Z}^n$,

$$\left\langle \mathbf{z}, \mathbf{y} \right\rangle = \mathbf{z}^T \mathbf{x} = \underbrace{\left(\mathbf{H}^+ \right)^T \mathbf{x}_1}_{\mathbf{z} \in \Lambda^{(D)}} \underbrace{\mathbf{H}\mathbf{x}_2}_{\mathbf{y} \in \Lambda} = \mathbf{x}_1^T \mathbf{H}^+ \mathbf{H} \mathbf{x}_2 = \mathbf{x}_1^T \mathbf{x}_2 \in \mathbb{Z} .$$

Furthermore, it is also possible to show that each point in the dual lattice can be written as an integer combination of the columns of $\mathbf{H}^{(D)}$. Let us focus on the case of full rank real matrices where $\mathbf{H}^{+} = \mathbf{H}^{-1}$. Denoting the rows of $\mathbf{H}^{-1}$ by $\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_n$, for any point $\mathbf{z} \in \Lambda^{(D)}$ it is possible to write

$$\begin{aligned}\mathbf{z}^T &= \mathbf{z}^T \mathbf{H}\mathbf{H}^{-1} \\ &= \underbrace{(\mathbf{z}^T \mathbf{h}_1)}_{\in \mathbb{Z}}\mathbf{r}_1 + \underbrace{(\mathbf{z}^T \mathbf{h}_2)}_{\in \mathbb{Z}}\mathbf{r}_2 + \cdots \underbrace{(\mathbf{z}^T \mathbf{h}_n)}_{\in \mathbb{Z}}\mathbf{r}_n,\end{aligned} \tag{11}$$

which shows that the point in the dual lattice is defined by a linear combination of the rows of $\mathbf{H}^{-1}$, i.e., a linear combination of the columns of $(\mathbf{H}^{-1})^T$. These arguments can be extended to the cases where the Moore-Penrose inverse is required and also to complex lattices.

One interesting relationship between the two bases is that

$$\left(\mathbf{H}^{(D)}\right)^T \mathbf{H} = \mathbf{I}, \tag{12}$$

which is equivalent to saying that $\left\langle \mathbf{h}_i, \mathbf{h}_j^{(D)} \right\rangle = \delta_{i,j}$, using the Kronecker delta.

The volumes of the primal and the dual lattice are related by $\mathrm{vol}\left(\Lambda_D\right) = \left(\mathrm{vol}(\Lambda)\right)^{-1}$ and their Gram matrices are related by $\mathbf{G}^{(D)} = \mathbf{G}^{-1}$.

Obviously, the dual of the dual lattice is the primal lattice itself. The geometry of the dual lattice is closely related to the geometry of the primal lattice. The connection is that each point in the n-dimensional dual lattice defines a family of parallel (n–1) dimensional hyperplanes onto which translates of a (n–1)-dimensional sublattice lie. The union of those planes captures all the points of the primal lattice. This means that the shortest vector in the dual lattice will define the most distant (n–1)-dimensional hyperplanes, whose union builds up the whole primal lattice. These hyperplanes can be interpreted as parallel layers and (as a consequence of being the ones furthest apart) are the densest ones in the lattice. In MIMO literature, the geometrical interpretation of the dual lattice as a tool for improving detection seems to have been first noticed in [15] (p. 2207) for sphere decoding, and then in [16] and [17], though it is also implied in the detector in [18] (p. 1944). From definition (8), in both Λ and $\Lambda^{(D)}$, the inner product between some given point $\mathbf{z}$ in the dual lattice and any vector in the primal lattice is always an integer, and therefore,

$$\begin{aligned}&\left\langle \mathbf{z}, \mathbf{x} \right\rangle \in \mathbb{Z}, \quad \mathbf{z} \in \Lambda^{(D)}, \mathbf{x} \in \Lambda \Leftrightarrow \\ &\Leftrightarrow \left\|\mathbf{z}\right\|\left\|\mathbf{x}\right\| \cos(\theta) = \left\|\mathbf{z}\right\| \mathrm{Proj}_{\mathbf{e}_z}(\mathbf{x}) \in \mathbb{Z},\end{aligned} \tag{13}$$

where $\overline{\mathbf{z}} = \mathbf{z} / \left\|\mathbf{z}\right\|$. It is then possible to define a family of parallel hyperplanes $\mathcal{P}(\nu)$, for $\nu \in \mathbb{Z}$, such that $\mathrm{Proj}_{\overline{\mathbf{z}}}(\mathbf{x}) = \left\|\mathbf{z}\right\|^{-1} \nu$. These are planes in dimension $n-1$, with a distance $d = \left\|\mathbf{z}\right\|^{-1}$ between them, as illustrated in Fig. 1. Remark: all vectors $\mathbf{a}_i$ in a given hyperplane have the same inner product with $\mathbf{z}$.

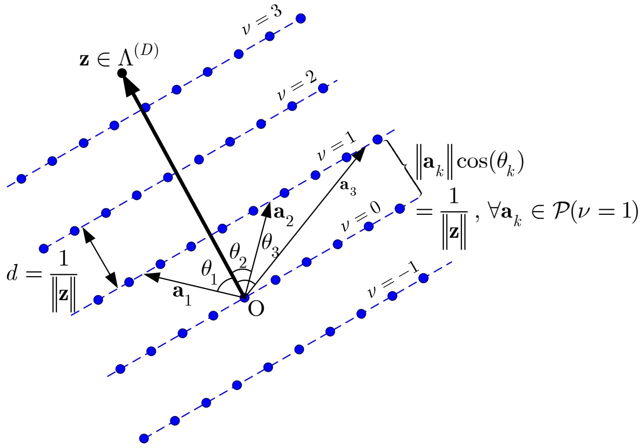


Fig. 1: A primal lattice in n dimensions as the union of translates of a sublattice and these translates lie on $(n-1)$-dimensional hyperplanes.

IV. Proposed Receiver

A. Successive Minima in the Dual Lattice

The dual lattice, as is the case of any lattice, has a shortest vector (which comes at least paired with its symmetrical vector) and a set of other successive minima. Because some of these vectors may be linearly dependent, the interesting definition of successive minima imposes independence. Hence, λ_i is the i^{th} *successive minimum* of a lattice if λ_i is the smallest real number that is the smallest radius of a sphere that contains i independent vectors, all with norms smaller or equal to λ_i. The shortest vector obviously has norm λ_1.

From Section III it is possible to conclude that the hyperplanes which are furthest apart from each other (and thus having the highest density of lattice points on them) are defined by the shortest vector in the dual lattice. This observation is essential to explain which layer must be detected first in a OSIC receiver. The selection of the next layer is determined by the same observation, applied now to the sublattice spanned by the matrix obtained after striking out from $\mathbf{H}$ the column generator preciously detected.

This approach to OSIC, pioneered in [17], leads to the previously known optimal ordering in V-BLAST detection [3] while offering an elegant explanation to it based on the geometric properties proven in Section III. This approach leads to the well-known performance of the original V-BLAST, however, accomplishing it without needing a matrix inversion associated with each layer to be detected, as is required in the original V-BLAST algorithm.

This paper proposes that not only the family of hyperplanes that are furthest apart are used, but also that other families of hyperplanes associated with some of the successive minima of the dual lattice should also be brought to use. Fig. 2 shows an example of two of those different partitions of a lattice defined by

$$\mathbf{H} = \begin{bmatrix} 3/7 & -2/7 \\ -1/7 & 3/7 \end{bmatrix} \quad \text{and with} \quad \mathbf{H}^{(D)} = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix},$$

associated with two different choices of vectors in $\Lambda^{(D)}$.

Consider the hyperplanes selected by the first L successive minima in $\Lambda^{(D)}$, i.e., $\lambda_i, \cdots \lambda_L$. Finding the shortest vector in a lattice is itself a NP-hard problem, which implies the same complexity for obtaining the L shortest ones. Nevertheless, if this is only required at a pre-processing stage, and not needed for each received vector, then using a sphere decoder is acceptable. While its complexity is exponential in the dimension of the lattice [12], this cost is only necessary whenever the channel changes, which is tolerable for slow fading channels.

SD is today one of the most used methods for solving the CVP in MIMO systems, despite having originally been presented as a technique for solving the shortest vector problem (SVP) by Finke and Pohst (c.f. [15]). In the following, we make use of these fundamental ideas to list all the points of a lattice inside a hypersphere of radius ρ centred in the origin, which for this reason only requires a very simple implementation of the SD such as the one given in [12]. We are interested in the set of lattice points spanned by $\mathbf{H} \in \mathbb{R}^{m \times n}$ which verify

$$\left\{ \mathbf{y} \in \mathbb{R} : \mathbf{y} = \left\| \mathbf{H} \cdot \mathbf{x} \right\|^2 \leq \xi^2, \quad \mathbf{x} \in \mathbb{Z} \right\}, \tag{14}$$

or, using the QR decomposition with $\mathbf{Q}$ orthogonal and $\mathbf{R}$ upper triangular, one can search the list of vectors $\mathbf{x}$ as

$$\left\{ \mathbf{Q}^{-1}\mathbf{y} \in \mathbb{R} : \mathbf{Q}^{-1}\mathbf{y} = \left\| \mathbf{R} \cdot \mathbf{x} \right\|^2 \leq \xi^2, \quad \mathbf{x} \in \mathbb{Z} \right\}. \tag{15}$$

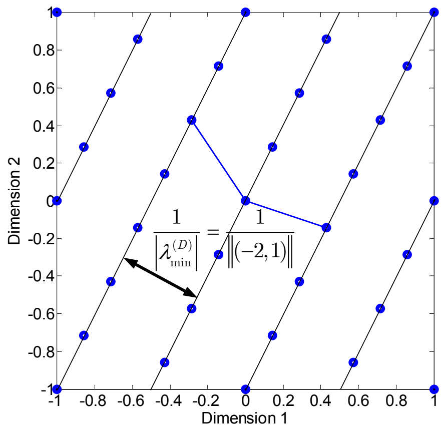
As $\mathbf{R}$ is upper triangular, the norms in (15) are

$$\sum_{i=1}^{m} \left(\sum_{j=1}^{n} \mathbf{R}_{i,j} \left(x_j \right) \right)^2 \leq \xi^2 , \tag{16}$$

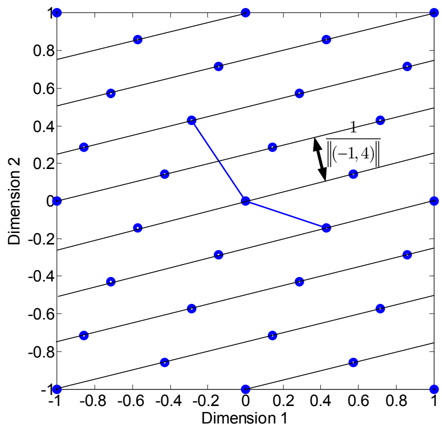
which allows the characteristic tree exploration of SD. Because one is interested in planes with different distances, not all the lattice points with $\left\| \mathbf{y} \right\| < \xi$ are a successive minima and they need to be expunged from the list. When centred at the origin, an implementation of SD such as that in [12], outputs a list of (column) vectors arranged as

$$\big[\underbrace{\mathbf{s}_1, \mathbf{s}_2, \cdots \mathbf{s}_{N/2-1}}_{N/2-1}, \mathbf{0}, \underbrace{\mathbf{s}_{1N/2+1}, \cdots \mathbf{s}_N}_{N/2-1} \big]$$

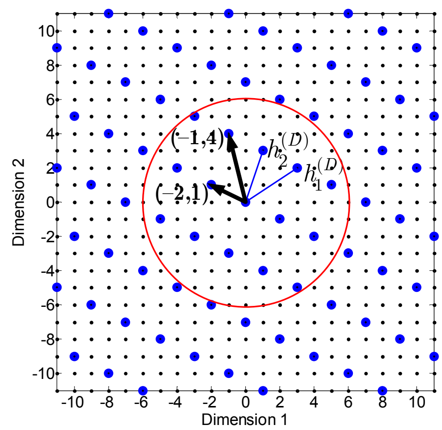
where $\mathbf{0}$ is the origin, which is always captured in the set for any $\rho > 0$ and N is the number of lattice points inside the sphere of radius ξ.



(a) Hyperplanes in the primal lattice associated with (–2,1) in the dual lattice.



(b) Hyperplanes in the primal lattice associated with (–1,4) in the dual lattice.



(c) Vectors selected in the dual lattice (black arrows).

Fig. 2: Identification of the hyperplanes in the primal lattice associated with a given vector in the dual lattice.

The two sides of the output around 0 have the same vectors up to their sign and therefore the selection of the first $N/2-1$ suffices. In addition to that selection, one will just take one vector for each distinctive norm, even if there are several linearly independent ones. This widens the range of different distances between hyperplanes. The resulting set of vectors in the dual will be dubbed *unique successive minima* (USM). This concept is depicted in Fig. 2 (c), where L=7 USM are found inside the sphere in $\Lambda^{(D)}$.

B. Projections Onto Hyperplanes

The L USM in the dual lattice are denoted by $\mathbf{v}_1^{(D)}, \mathbf{v}_2^{(D)}, \cdots, \mathbf{v}_L^{(D)}$. Naturally, the unit vectors which are orthogonal to the families of hyperplanes are

$$\overline{\mathbf{v}}_i^{(D)} = \mathbf{v}_i^{(D)} \Big/ \left\| \mathbf{v}_i^{(D)} \right\| \tag{17}$$

and we further define the vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_L$, each one respectively collinear with $\mathbf{v}_i^{(D)}$, but forced to have norm d= $\left\| \mathbf{v}_i^{(D)} \right\|^{-1}$, as has been suggested in Fig. 1. Hence, from (17), these vectors should be $\mathbf{v}_i = \mathbf{v}_i^{(D)} \Big/ \left\| \mathbf{v}_i^{(D)} \right\|^2$.

The projections of the received vector (i.e., the target in the CVP) onto a family of $\mathcal{P}_{\mathbf{v}_i}(\nu)$ hyperplanes generate the set of projection points

$$\mathbf{y}_p(\mathbf{v}_i, \nu) = \mathbf{y} + \underbrace{\left[Q_{\mathbb{Z}}\left(\frac{\langle \mathbf{y}, \mathbf{v}_i \rangle}{\|\mathbf{v}_i\|^2} \right) - \frac{\langle \mathbf{y}, \mathbf{v}_i \rangle}{\|\mathbf{v}_i\|^2} \right]}_{\Omega} \mathbf{v}_i + \nu\, \mathbf{v}_i. \tag{18}$$

where $\nu \in \mathbb{Z}$ and $Q_{\mathbb{Z}}(\cdot)$ denotes rounding to $\mathbb{Z}$. This way, in the case of no noise in the system, the number Ω will *always* be an integer, indicating in which hyperplane the lattice lies, for each family $\mathcal{P}_{\mathbf{v}_i}$. Hence,

$$\mathbf{y}_p(\mathbf{v}_i, \nu) = \mathbf{y} + \left(\Omega(\mathbf{v}_i) + \nu \right) \mathbf{v}_i. \tag{19}$$

C. List of Candidate Solutions

Fixing L as the number of USM, and setting $\nu_{\max}$ as the maximum value of ν that will be explored, it is possible to obtain a set C consisting of the candidate vectors obtained from

$$\mathbf{y}_i^{(C)} = \mathbf{H}\, Q_{\mathcal{A}}\left(\mathbf{H}^{+} \mathbf{y}_p(\overline{\mathbf{v}}_i, \nu) \right) \quad, i = 1, 2, \cdots, |C|, \tag{20}$$

where $Q_{\mathcal{A}}()$ denotes quantization to the alphabet $\mathcal{A}$ used in each dimension using the Moore-Penrose pseudo-inverse. The total number of candidates considered in (22) is given by the number of families of hyperplanes considered (i.e., the number of USM inside a sphere) multiplied by the number parallel of hyperplanes considered (the closest one and the adjacent ones with non-zero index ν):

$$|C| = L \cdot (2 \cdot \nu_{\max} + 1). \tag{21}$$

This amounts to performing zero-forcing detection not only to $\mathbf{y}$ but also to the set of all projections onto $\mathcal{P}_{\mathbf{v}_i}(\nu)$. The solution to the MIMO detection problem is then obtained by applying the maximum likelihood principle to all the vectors in the set of candidates C

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y}_i^{(C)} \in C} \left\{ \left\| \mathbf{y} - \mathbf{y}_i^{(C)} \right\|^2 \right\}. \tag{22}$$

The requirement for a correct detection is now not restricted to having the received $\mathbf{y}$ inside the (possibly narrow) fundamental decision region associated with the basis $\mathbf{H}$; it suffices that one of the projections lies inside it. This concept is depicted in Fig. 3, which shows the projections of $\mathbf{y}$ onto the three densest families of hyperplanes with lattice points. The circle lines in Fig. 3 correspond to the nine projections that are generated, 3 in each family of hyperplanes.

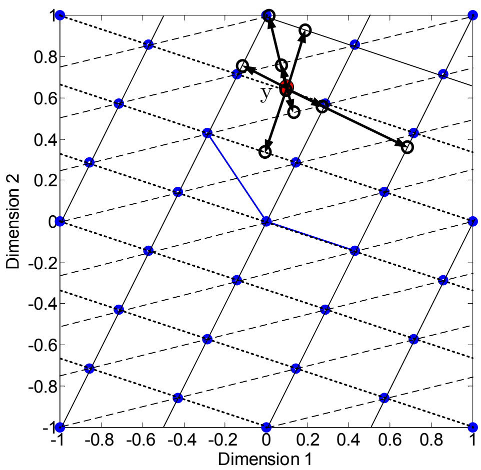


Fig. 3: Dual-lattice-aided generation of candidate solutions considering $\nu_{\max}$=1 and considering L=3 families of hyperplanes: the two families in Fig. 2 and also the family associated with the dual vector $h_2^{(D)} = (1, 3)$.

V. Simulation Results

The performance of the dual-lattice-aided (DLA) receiver is assessed in terms of the (complex) symbol error rate (SER) with L=4n, for 3×3 and 4×4 antennas (i.e., lattices with n=6 and n=8 dimensions, i.e., with L=24 and L=32 respectively) using 64-QAM. The results are shown in Fig. 4 and Fig. 5. These figures also include the following traditional receivers: ZF, MMSE, OSIC, LRA using LLL pre-processing with ZF and also with OSIC-ZF, besides ML (using SD). In both cases OSIC is outperformed. The DLA receiver exhibits better performance than LRA in the low SNR regime, but because LRA achieves the full diversity of the channel [9] the SER of LRA eventually drops below the one of the proposed algorithm. It has also been found that, as expected, when either L or $\nu_{\max}$ decreases, the performance degrades.

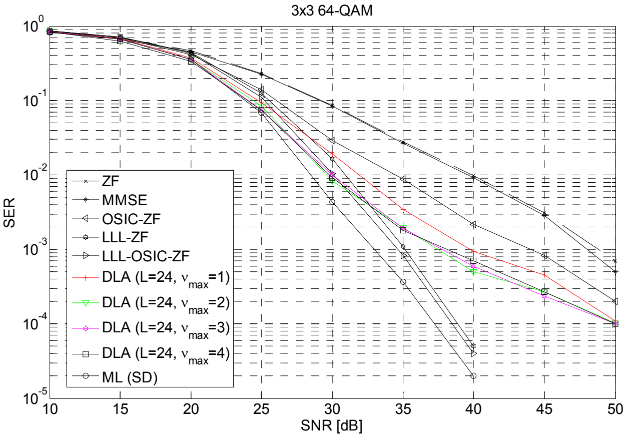


Fig. 4: SER vs SNR for 3×3 antennas and 64-QAM.

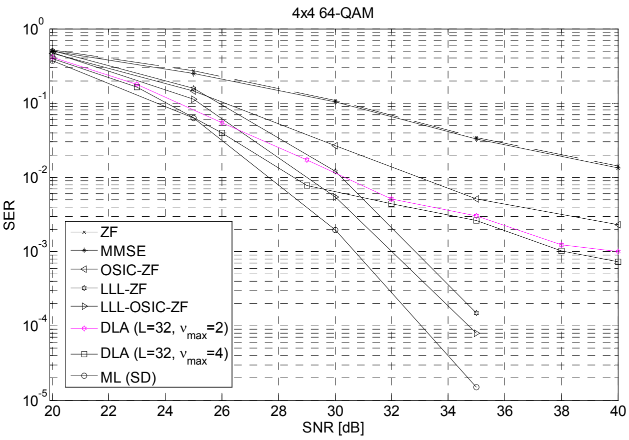


Fig. 5: SER vs SNR for 4×4 antennas and 64-QAM.

VI. Conclusions

This paper starts by clarifying the geometric relation between the primal and the dual lattice, which is often overlooked in the literature. Then, a simple receiver for spatial multiplexing is proposed, leading to a SER that outperforms OSIC. While OSIC requires a matrix inversion when detecting each layer, the proposed DLA receiver generates a list of candidates by means of a one-shot matrix product that projects the target point onto families of hyperplanes surrounding the target point. Subsequently, the best one of them is selected by applying ZF. This approach leads to significant gains (10dB and 4dB respectively for the 3×3 and 4×4 configurations) with respect to OSIC. This is achieved with a reasonably low number of candidates (e.g., with 3×3 antennas, only $|C| = 24 \cdot (2+1) = 72$ candidates are needed because $\nu_{\max} = 2$ suffices). The projection matrix is obtained during the pre-processing stage by means of a naive SD that finds short vectors in the dual lattice, but this step is only required when the channel information at the receiver needs to be updated.

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