Limits for CPM signals representation by Walsh functions

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Abstract – This paper explores the feasible limits for complexity reduction of a very simple front-end block for the calculus of phase transition metrics on a continuous phase modulation (CPM) receiver. A quasi-optimum receiver of very low complexity is attained by splitting the function of the optimum receiver bank filters in two blocks: calculus of projections coefficients on a low dimensional space of Walsh functions followed by simple matrix calculus. A sequence detection algorithm follows this block. The presented approach enables the reduction of the matched filters or correlators to just two integrators, regardless of the CPM scheme. Research on the reduction limits of the space dimension is conducted using catastrophic M-ary CPM schemes, taking advantage of their very low number of phase states. Performance of 1REC $h = 1/2$ 16-ary scheme is for the fist time presented. A rule is defined concerning the number of Walsh functions that must be used. That outcome proves to be valid for two CPM schemes of high power gain. The receiver is tested under additive white gaussian noise (AWGN).

Index terms – Continuous phase modulation (CPM), metrics calculus, Walsh functions

I. Introduction

Continuous phase modulation (CPM) signals have constant amplitude and so they are a good solution for systems requiring insensitivity to non-linear amplitude amplification. Their phase continuity allows good spectral performance and implies a code gain due to the inherent memory effect. These properties have motivated the common use of GMSK (gaussian minimum shift keying), which is a simple member of the CPM family, in widespread use systems such as GSM/DCS, PCS, DECT, CT2 and Bluetooth. The use of other CPM schemes more spectrally efficient and better power efficient was restrained owing to excessive detection complexity [1]. The number of analogue matched filters (or correlators) needed is often unbearable for practical implementation. The number of phase states to be detected can very large as well. Conception of simple receivers is nowadays a main concern within CPM research. This paper shows that Walsh functions can generate a space where it is possible to find signals close to the original CPM signals. Digital signal processing (DSP) allows fast matrix calculus using both received and stored signals.

II. CPM formatting and performance

Every CPM signals can be expressed in the form:

$$s(t, \tilde{\mathbf{a}}) = \sqrt{2E_s/T_s}\cos(\omega_c t + \varphi(t, \tilde{\mathbf{a}}) + \varphi_0) \qquad (1)$$

The carrier frequency is f_c, where $\omega_c = 2\pi f_c$, φ_0 is the arbitrary initial phase and E_s is the energy per symbol, related with the bit energy by $E_s = \log_2(M)\cdot E_b$. Channel symbols are $\gamma_i \in \{\pm1, \pm3, \cdots, \pm(M-1)\}$, forming the M-ary sequence $\boldsymbol{\gamma}$. Each symbol γ_i carries $\log_2(M)$ bits as a result of a natural mapping of the information bits stream $\boldsymbol{\alpha}$. The information carried by N_S channel symbols is keyed in signal's phase:

$$\varphi(t, \tilde{\mathbf{a}}) = 2\pi\, h \sum_{i=0}^{N_s} \gamma_i q(t - iT_s) \qquad (2)$$

A constant modulation index, $h = p/q$, is considered, where p and q are integers with no common factors. (h is rational in order to have a finite number of phase states.) Phase transition pulse shape, $q(t)$, affects phase transitions shape during L symbols. However, its effect remains until the end of the transmitted sequence. $q(t)$ is defined by the frequency pulse $g(t) = \int_{-\infty}^{t} g(\tau)d\tau$. The normalisation $q(t) = \int_0^{\infty} g(\tau)d\tau = 1/2$ is applied so that the maximum phase transition during a symbol time, T_s, is $h\cdot(M-1)\cdot p$. Different frequency pulses define different CPM families. The most common are: LREC, LRC (L is the variable mentioned above) and GMSK [1,2]. LREC is defined by $g(t) = \mathrm{rect}[t/(LT_s)]/2$, where $\mathrm{rect}(t) = 1$ for $-1/2<|t|<1/2$ and zero elsewhere. Schemes with 1REC pulses are also known as CPFSK (continuous phase frequency shift keying). A smother $g(t)$ such the named LRC (raised cosine pulse shaping) usually conducts to narrower bandwidth than the ones given by LREC pulses.

In order to evaluate CPM power performance one uses the *minimum normalised squared euclidean distance* (MNSED) between two signals transporting sequences $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma}'$:

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(3) $$d^2_{\min}(\tilde{a},\tilde{a}') = 1/(2E_s) \min_{\tilde{a},\tilde{a}':\tilde{a}\neq\tilde{a}'} \int_0^\infty [s(t,\tilde{a}) - s(t,\tilde{a}')]^2 dt$$

Bit error rate (BER) is given by (e.g. [1])

(4) $$P_b \approx C \cdot Q\left(\sqrt{d^2_{\min}\frac{E_b}{N_0}}\right) \approx Q\left(\sqrt{d^2_{\min}\frac{E_b}{N_0}}\right)$$

C is a constant ≈1 for most schemes (and 2 for MSK). $Q(x)$ is the area under the unit variance gaussian distribution in $[x, \infty]$. Power efficiency comparisons can be made from (4) merely by $d^2_{\min}$ knowledge or converting it to a gain relative to MSK, being $G = 10 \cdot \log_{10}(d^2_{\min}/2)$ [dB].

Bandwidth is usually given in terms of $B_\varepsilon T_b$ where B_e is the bandwidth that enclosures ε% of all transmitted power. $T_b = T_s/\log(M)$ is the bit interval; bit rate is $R_b = 1/T_b$. For MSK $B_{99.0}T_b = 1.2$. Spectrum efficiency is thereby $\zeta = 1/(B_\varepsilon T_b) = R_b/B_\varepsilon$. By reducing h, phase transitions get smother, tightening bandwidth, but that also forces MNSED to decrease due to the greater similitude among transitions during each T_s interval. A greater M enhances simultaneously spectrum and power behaviour at a cost of boost in complexity.

III. Optimum detection

To obtain metrics for each one of the Ξ phase transitions the optimum CPM receiver requires 2Ξ matched filters (or equivalent correlators), one for each branch I and Q. Metrics have to be calculated for all transitions $\tau_{I,b} \in \{\tau_{I,1}, \tau_{I,2}, \cdots, \tau_{I,\Xi}\}$ and all $\tau_{Q,b} \in \{\tau_{Q,1}, \tau_{Q,2}, \cdots, \tau_{I,\Xi}\}$. Considering $n(t)$ additive white gaussian noise (AWGN), after baseband conversion one gets the $y(t) = s(t, \gamma) + n(t)$. I and Q metrics for $b = 1, 2, \cdots, \Xi$, are then

(5) $$\Lambda_i(b) = \int_{T_s} y_{I,i}(t)\tau_{I,b}(t)\,dt + \int_{T_s} y_{Q,i}(t)\tau_{Q,b}(t)\,dt = \Lambda_{I,i}(b) + \Lambda_{Q,i}(b).$$

In more detail, for the same b, the branch metrics are

(6a) $$\Lambda_{I,i}(b) = \int_{iT_s}^{(i+1)T_s} y(t)\cdot\cos[2\pi h\gamma_b q(t)]dt$$

(6b) $$\Lambda_{Q,i}(b) = \int_{iT_s}^{(i+1)T_s} y(t)\cdot\sin[2\pi h\gamma_b q(t)]dt\,.$$

Finally, having all the metrics, the problem is solved by a maximum likelihood sequence detector (MLSD). The detection complexity of CPM schemes is measured in terms of Ξ and the total number of states, being that number $S = q\times M^{L-1}$, for even p and $S = q\times M^{L-1}$ for odd p. In the case of full response systems ($L = 1$), S corresponds to the number of physical phase states.

The number of phase transitions is therefore $\Xi = S\times M$. For this reason the number of 2Ξ filters becomes intolerable for high M and/or weak h.

As a result, MLSD must search for the sequence having maximum cumulative metric given by the inner product

(7) $$\Lambda_i(b) = \langle y(t,\gamma_i), \tau_b(t)\rangle\,.$$

IV. Projections and metric calculus

Metrics are calculated on a F-dimensional Walsh space, generated by F Walsh functions [3] of order k denoted as $w_{F,n}(t)$; $n = 1, 2, 3, \cdots, F = 2^k$; each one with $F = 2^k$ symbols, being them $w_{F,n}[j]$, $j = 0, 1, \cdots, F{=}2^k$, for $k \in Z^+$:

(8) $$\tilde{w}_{F,n}(t) = \frac{1}{\sqrt{T_s}}\sum_{j=0}^{F-1} w_{F,n}[j]\,\mathrm{rect}\left[\frac{t - (T_s/(2F)) - j(T_s/F)}{T_s/F}\right]$$

with $n = 0, 1, \cdots, F-1 = 2^u - 1$, $u\ k \in Z^+$, and $\mathrm{rect}(t) = 1$ for $<|t|<1/2$ and zero elsewhere. Symbols $w_{F,n}[j]$ $k \in \{-1, +1\}$ and are defined by a recursive method that builds *Walsh-Hadamard matrixes* [3]. Applying (7), it can be proved that the b^{th} metric in the Walsh space when applying a MLSD criterion is given by

(9) $$\Lambda_i(b) = \left[\sum_{n=1}^{F}\int_{iT_s}^{(i+1)T_s} y(t)\cdot\tilde{w}_{F,n}(t)\,dt - \int_{iT_s}^{(i+1)T_s} s(t,\gamma_b)\cdot\tilde{w}_{F,n}(t)\,dt\right]^2$$

for $b = 1, 2, \cdots, \Xi$. Metric calculus is made merely using the projection of the received baseband signal $y(t)$ into the Walsh space. Those projections coefficients are

(10) $$c_{i,n} = \frac{1}{\sqrt{T_s}}\int_{iT_s}^{(i+1)T_s} y(t,\gamma_i)\cdot w_{F,n}(t - iT_s)\,dt$$

From (9), the transition metrics are

(11) $$\Lambda_i(b) = \frac{1}{\sqrt{T_s}}\sum_{n=1}^{F}\left|c_{i,n} - c_{b,n}\right|^2, \text{ for } b = 1, 2, \cdots, \Xi,$$

where $c_{i,n}$ are the projection coefficients of the transition during symbol interval i, as given by (10), and $c_{b,n}$ are projection coefficients of the b^{th} transition belonging to the set of Ξ possible ones. Using the projection vectors and (7), (11) becomes

(12) $$\Lambda_i(b) = \frac{1}{\sqrt{T_s}}\sum_{n=1}^{F} c_{i,n}c_{b,n}\,,\quad b = 1, 2, \cdots, \Xi.$$

These coefficients can be easily determined by:

(13) $$c_{i,n} = \frac{1}{\sqrt{T_s}}\sum_{j=0}^{F-1} w_{F,n}[j]\int_{(i+j/F)T_S}^{(i+(j+1)/F)T_S} y(t,\gamma_i)\,dt\,.$$

Moreover, each integrator does not need to be dumped at the end of every T_s/F sub-interval. By sampling the continuous integration it is possible do know the partial integration values making

$$c_{i,n} = \frac{1}{\sqrt{T_s}} \sum_{j=1}^{F} w_{F,n}[j] \left[\left(\int_{iT_s}^{t} y(t,\gamma_i)\, dt \right) \Bigg|_{t=j\frac{T_s}{F}} - \left(\int_{iT_s}^{t} y(t,\gamma_i)\, dt \right) \Bigg|_{t=(j-1)\frac{T_s}{F}} \right], \quad iT_s < t < (i+1)T_s . \tag{14}$$

The calculation on (14) only requires two integrators, a sampling procedure and a calculus unit, independently of the CPM scheme.

The vector of Ξ metrics is the column vector

$$\ddot{\mathbf{E}}_i = \left[d_i^2(1)\ d_i^2(2)\ \cdots\ d_i^2(b)\ d_i^2(b+1)\ \cdots\ d_i^2(\Xi) \right]^{\mathrm{T}} . \tag{15}$$

The received i^{th} transition has a description on the Walsh space given by the projection vector

$$\mathbf{c}_i = \left[c_{i,1}\ c_{i,2}\ \cdots\ c_{i,n}\ \cdots\ c_{i,F} \right], \quad i=1, 2, \cdots, N_S. \tag{16}$$

Each possible transitions is stored on similar vectors:

$$\mathbf{c}_b = \left[c_{b,1}\ c_{b,2}\ \cdots\ c_{b,n}\ \cdots\ c_{b,F} \right], \quad b=1, 2, \cdots, \Xi. \tag{17}$$

Coefficients, $c_{b,n}$, can be determined and memorized in advance. Vectors $\mathbf{c}_b$ can form matrix $\mathbf{C}$ of dimensions $\Xi \times F$

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_b \\ \vdots \\ \mathbf{c}_\Xi \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} & \cdots & c_{1,F} \\ c_{2,1} & c_{1,2} & \cdots & c_{1,n} & \cdots & c_{1,F} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{b,1} & c_{b,2} & \cdots & c_{b,n} & \cdots & c_{b,F} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{\Xi,1} & c_{\Xi,2} & \cdots & c_{\Xi,n} & \cdots & c_{\Xi,F} \end{bmatrix} . \tag{18}$$

Having $\mathbf{C}$, the incremental distance vector (metrics) is

$$\ddot{\mathbf{E}}_i = \begin{bmatrix} d_i^2(1) \\ d_i^2(2) \\ \vdots \\ d_i^2(b) \\ \vdots \\ d_i^2(\Xi) \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} & \cdots & c_{1,F} \\ c_{2,1} & c_{1,2} & \cdots & c_{1,n} & \cdots & c_{1,F} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{b,1} & c_{b,2} & \cdots & c_{b,n} & \cdots & c_{b,F} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{\Xi,1} & c_{\Xi,2} & \cdots & c_{\Xi,n} & \cdots & c_{\Xi,F} \end{bmatrix} \cdot \begin{bmatrix} c_{i,1} \\ c_{i,2} \\ \vdots \\ c_{i,n} \\ \vdots \\ c_{i,F} \end{bmatrix} = \mathbf{C} \cdot \mathbf{c}_i^{\mathrm{T}}, \quad i=1, 2, \cdots, N_S. \tag{19}$$

conducting to the column vector containing the Ξ metrics.

V. Test schemes

In order to research the behaviour of the receiver we have used the $h = 1/2$ full response M-ary schemes presented in Table I (MSK on the first line), taking advantage of their very low number of states ($S = 4$). Those simple schemes happen to be catastrophic, that is, their MNSED has a local mean for the used $h = {}^1/_2$, bein the real $d^2_{\min}$ very distant from its upper bund [2]. That concerns only to the MLSD block and should not influente the research on the metric calculus using the given CPM space approximation.

From [2,4,5] we point out two optimum full response mono-h CPM schemes also characterized in Table 1.

Table 1.

h	M	S	$B_{99.0}T_b$	$d^2_{\min}$	G [dB]	Ξ	2Ξ
1/2	2	4	1.20	2.0	0	8	16
	4	4	1.30	2.0	0	16	32
	8	4	1.55	3.0	1.76	32	64
	16	4	(a)	(a)	(a)	64	128
9/20	4	40	1.18	3.60	2.56	160	320
	8	40	1.40	5.40	4.31	320	640

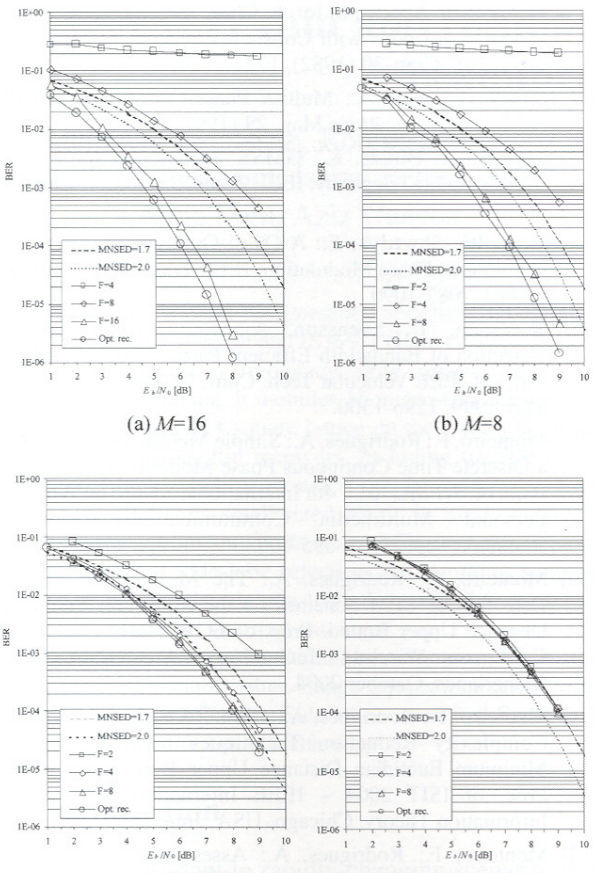
Table 1 positions where "(a)" is found are not available; they are obtained in Section VI. The selected schemes of $h = 0.45 = 9/20$ are the best 4-ary and 8-ary CPFSK schemes in terms of power gains within the regiom of used spectral efficiencies which preserve an acceptable number of states ($S = 40$). These two schemes of $h = 0.45$ share another interesting feature: they are examples of rare schemes with a MNSED coincident with their upper bound curves (determined by [1,2]).

VI. Results

Results for performance in terms of bit error rate (BER) for detection under AWGN are depicted in Figures 1 and 2.

The results for the optimum reception of 1REC, $h = 1/2$, $M = 16$ plotted in Figure 1 (a) were achieved for the fist time as a result of the presented research: a gain of ≈3 dB can be detected, corresponding to $d^2_{\min} \cong 4$ (half the bound value for $d^2_{\min}$ calculated by [1].

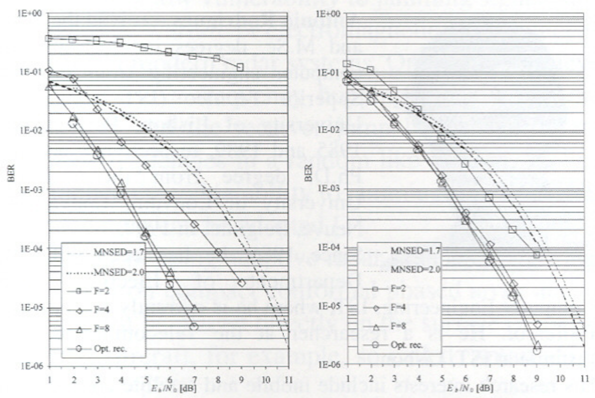
For better power gain assessment all figures include both BER curve for ideal antipodal modulation ($d^2_{\min} = 2$) and BER curve associated to $d_{\min} = 7.1$, proposed by [6] to describe real MSK. It can be seen that that an increasing number of Walsh functions, F, is required



(a) M=16 (b) M=8

(c) M=4 (d) M=2 (MSK)

Fig. 1. Effect of the Walsh space dimension for h = 1/2, 1REC schemes.



(a) M=8 (b) M=4

Fig. 2. Effect of the Walsh space dimension for optimum = 9/20, 1REC schemes.

when a greater M scheme is considered. For all tested schemes a number of Walsh functions equal to M (i.e. $F = M$) assures near optimum performance. For $F > M$ no significant gains were detected. A decreasing value of F implies an abrupt loss.

This method was firstly proposed by [7] for schemes 3RC (partial response scheme) with $M = 2$ and $M = 4$ and a varying h. It was also seen there that power penalty decreases for small modulations indexes (which are the interesting ones in terms of bandwidth). This fact is coherent with the greater smoothness on the phase transitions of low h schemes (less abrupt signals) which brings CPM signals closer to the Walsh space. Those results from [7] also show that, for those schemes, even when $F = 2$ the power penalty is always <0.5 dB. For $F = 4$ and for F=8 the loss is always <0.4 dB and <0.1 dB respectively for schemes having h<0.7. These impressive results were also verified on other partial response quaternary schemes with GMSK pulses with $L = 5$ and $L = 6$, and $h = 1/4$, 1/5 and 1/6 [8].

From the attained results it can be said that the rule $F \geq M$, found out for the simple but catastrophic schemes, can be extrapolated to the interesting schemes of $h = 9/20$, presented in section V, which were also tested under AWGN. More, simulations for the optimum receiver confirms the expected gains showed in Table 1 for these h=9/20 schemes: $G \approx 2.6$ dB for $M = 4$ and $G \approx 4.3$ dB for $M = 8$. When applying the Walsh space to the optimum $h = 9/20$ schemes one gets for $M = 8$ with $F = 8$ a BER curve as close as 0.2 dB from the optimum detection curve, as seen in Figure 2 (a). For $M = 4$ with $F = 4$ the power loss is less than 0.1 dB - Figure 2 (b). Also notice that $h = 0.45$ assures smother transitions than $h = 0.5$.

VII. Conclusions

The first results of [7] where extended in this paper and some patterns were found to predict the error robusteness of receivers using this low complexity front-end which completes CPM metrics based on a Walsh space. The good approximation of CPM signals by such simple functions is justified by the fact that CPM signals are inherently narrowband signals.

By assessing bit error rate performance with different Walsh space dimensions it was found that similar patterns occur for different M-ary schemes. A decrease on the order of the used set of Walsh functions degrades performance, being this effect more important for schemes with a higher M.

It was established that near-optimum performance is attained when using a Walsh space dimension as small as M. The use of a superior number of functions permits little performance improvement. The rule is valid at least for schemes with $h \leq 0.5$ (with are the most interesting ones in terms of bandwidth) and M ≤ 16 (the feasible ones in terms of complexity for MLSD) and proved to remain valid when applied to full response

optimum gain schemes (equal to the local power upper bound).
As a particular a case, it was showed that MSK can be detectable in a quasi-optimum manner just sampling twice a continuous integrator during each symbol interval.
The metric calculus made with this simple technique can still be applied when using symmetry relations to derive metrics among different quadrants [9]. Its joint application with both [9] and the use of the M-algorithm ruled by the results in [10], enabled the definition of a very low quasi-optimum receiver analysed in [11,12] using the optimum schemes selected and tested here.

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